Efficient MLFMA Analysis of Coupled Rectangular and Circular Apertures in a Finite Screen

I. Rullhusen, and F. Arndt

Microwave Department, University of Bremen, Otto-Hahn-Allee NW1, D-28359 Bremen, Germany
fritz.arndt@physik.uni-bremen.de

***Abstract*— A multilevel fast multipole algorithm (MLFMA) is introduced for the efficient analysis of coupled rectangular and circular apertures of different size within a finite conducting screen. The electric field integral equation (EFIE) is written in terms of mixed potential formulation. Aperture waveguide modal eigenvectors and Rao-Wilton-Glisson (RWG) functions for triangular patches are utilized as basis functions for the magnetic and electric surface current densities, respectively. For the resulting method-of-moments (MoM) solution, a MLFMA is formulated which includes the modal excitations of the coupled rectangular and circular apertures. This leads to significantly reduced computational complexity for these kinds of problems. The method is verified by available measurements and reference calculations.**

I. INTRODUCTION

Due to their complexity, coupled rectangular and circular apertures pose a considerable challenge for numerical solutions of their electromagnetic problems, in particular when higher-order modes and finite screens have to be taken rigorously into account. Since all important features, like overall modal scattering parameters and radiation characteristics are influenced by mutual coupling effects between elements, their accurate analysis is essential. Method-of-moment (MoM) solutions of coupled apertures are well-known [1] – [6], which are mainly based on the assumption of infinite screens [1] – [5]; an UTD approach is used in [6] adding edge-diffracted rays at finite screens.

The commonly applied iterative solvers for the involved matrix equations of MoM solutions require the performance of a matrix-vector product, which is usually the bottle-neck in such computations [7] of N unknowns due to the required $O(N^2)$ operations. Hence, for large scale problems like for coupled apertures, particularly when large finite metal plates are taken into account, it is imperative to reduce the complexity of the involved solution algorithm.

Fast integral solvers based on the fast multipole method (FMM) [7] - [9], and more recently on the multilevel fast multipole algorithm (MLFMA) [10] – [12] have proven to be well appropriate for large scale scattering problems. The MLFMA allows a matrix vector product to be effected in $O(N \log N)$ operations [10] – [12] for matrix equations resulting from the MoM. The MLFMA has mainly been

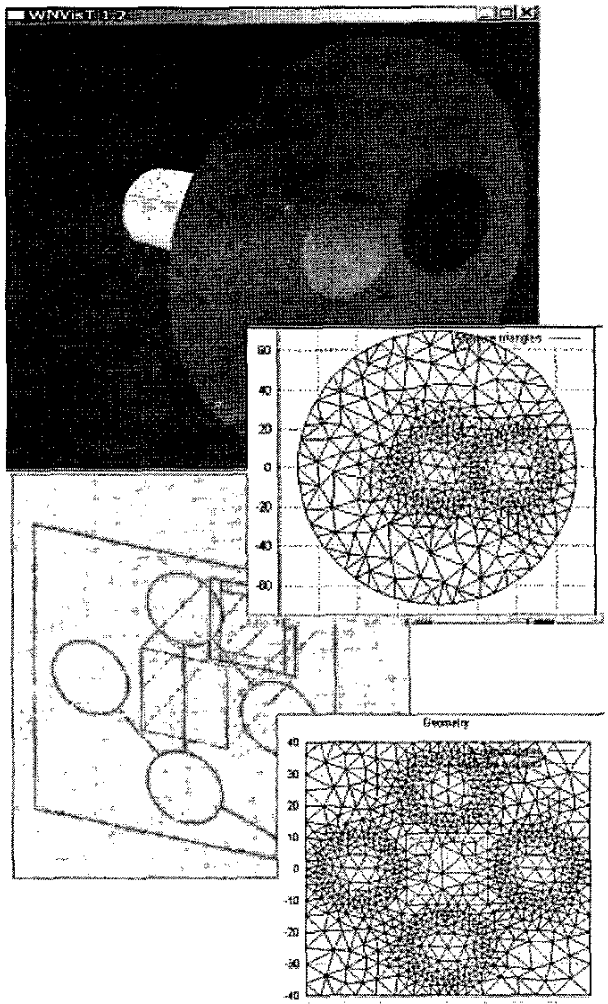

Fig. 1. Examples of coupled apertures in a finite screen

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restricted to scattering [7] – [11] and low-frequency circuit problems [12], so far. A MLFMA solution for a single ridged horn has been presented in [13]. Very recently, first applications to coupled rectangular apertures have been shown in [14].

This paper extends the MLFMA solution to the rigorous analysis of coupled rectangular and circular waveguide apertures of different size in a finite rectangular or circular screen, Fig. 1. The electric field integral equation (EFIE) is advantageously formulated in mixed potential form in order to reduce the order of singularities. Radiation patterns and S-parameters of typical structures are calculated by applying the MLFMA and compared with reference solutions or available measurements.

II. THEORY

For finite structures of ideal conductivity (Fig. 1) with apertures, the EFIE is formulated in terms of electric $\mathbf{J}_s$ and magnetic $\mathbf{M}_s$ surface current densities in the usual way

$$\mathbf{M}_s + \hat{n} \;\; \mathbf{E}^J\{\mathbf{J}_s\} + \hat{n} \;\; \mathbf{E}^M\{\mathbf{M}_s\} = -\hat{n} \;\; \mathbf{E}^{inc} \qquad (1)$$

where $\mathbf{J}_s$ are expanded in Rao-Wilton-Glisson (RWG) basis functions $\vec{f}_i$, and for $\mathbf{M}_s$ the normalized modal eigenvectors $\vec{g}_i = \mathbf{e}_i \;\; \hat{n}$ of the apertures are chosen [15]. The EFIE (1) is scalar multiplied by $\hat{n} \;\; \vec{f}_i$ and integrated over the corresponding area of the basis functions, as well as scalar multiplied by $\vec{g}_i$ and integrated over the aperture surfaces. This yields the linear equation system

$$\begin{pmatrix} \mathbf{Z}_1 & \mathbf{T}_1 \\ \mathbf{Z}_2 & \mathbf{T}_2 \end{pmatrix} \cdot \begin{pmatrix} \mathbf{J} \\ \mathbf{b} \end{pmatrix} = \begin{pmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \end{pmatrix} . \qquad (2)$$

The matrix elements are written in their mixed potential form. For $\mathbf{Z}$ we obtain

$$(\mathbf{Z}_1)_{ij} = -j\omega \iint_{S_i} \vec{f}_i \cdot \mathbf{A}_J\{\vec{f}'\}\, dS - \frac{1}{j\omega} \iint_{S_i} (div_s \vec{f}_i)\; \phi_J \; \{div_s' \vec{f}_i'\}\, dS \qquad (3)$$

$$(\mathbf{Z}_2)_{ij} = +j\omega \iint_{S_{Ap_i}} \vec{e}_i \cdot \mathbf{A}_J\{\vec{f}'\}\, dS + \frac{1}{j\omega} \iint_{S_{Ap_i}} (div_s \vec{e}_i)\; \phi_J \; \{div_s' \vec{f}_i'\}\, dS - \frac{1}{j\omega} \sum_i \oint_{\partial S_i} \left(\mathbf{e}_i \phi_J \{div_s' \vec{f}_i'\}\right) \cdot \hat{u}\, dl \qquad (4)$$

with the potentials

$$\mathbf{A}_J\{\vec{f}_j'\} = \iint_{S_j} G_{AJ} \cdot \vec{f}_j'\, dS'$$
$$\phi_J\{div_s' \vec{f}_j'\} = \iint_{S_j} G_{\phi J} \cdot div_s' \vec{f}_j'\, dS' \qquad (5)$$

where G are the corresponding Green's functions.

The $\mathbf{T}$ and $\mathbf{U}$ elements are given by the corresponding basis functions and excited or impressed electric fields on the apertures, respectively

$$(\mathbf{T}_1)_{ij} = -\iint_S \vec{f}_i \cdot \mathbf{e}_j dS + \iint_S \vec{f}_i \cdot \mathbf{E}^M\{\vec{g}_j'\}\, dS$$
$$(\mathbf{T}_2)_{ij} = -\iint_{S_{Ap}} \mathbf{e}_i \cdot \mathbf{e}_j dS - \iint_S \mathbf{e}_i \cdot \mathbf{E}^M\{\vec{g}_j'\}\, dS \qquad (6)$$

$$(\mathbf{U}_1)_i = -\iint_S \vec{f}_i \cdot \mathbf{E}^{inc} dS$$
$$(\mathbf{U}_2)_i = -\iint_{S_{Ap}} \mathbf{e}_i \cdot \mathbf{E}^{inc} dS \qquad (7)$$

$\mathbf{J}$, $\mathbf{b}$ in (2) are the expansion coefficients for the RWG and modal eigenvector basis functions, respectively.

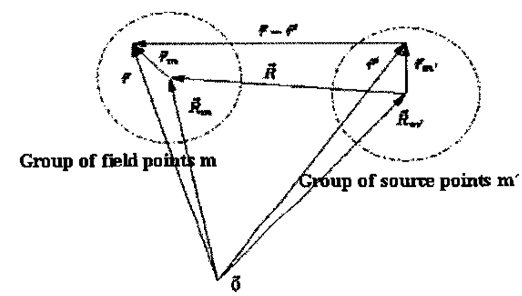

Fig. 2. Geometrical relations for source and field points used in the MLFMA equations

For the MLFMA, the matrices are separated according to the principle [9] – [14]

$$\mathbf{Z} = \mathbf{Z}^{MoM} + \mathbf{Z}^{FMM} \qquad (8)$$

where the 'near-neighbor' part $\mathbf{Z}^{MoM}$ is solved directly by the standard MoM with integrations only along the near neighbor range. For the 'far-neighbor' part $\mathbf{Z}^{FMM}$ the MLFMA yields

$$\left(\mathbf{Z}_1^{FMM}\right)_{ij} = jk \frac{Z_F}{4\pi} \frac{jk}{4\pi} \iint T_\infty\left(kR; \hat{k} \cdot \hat{R}\right) \cdot \iint_{S_i} e^{-jk\hat{k}\cdot\vec{r}_m} \left(\mathbf{I} - \hat{k}\hat{k}\right) \cdot \vec{f}_i dS \cdot \iint_{S_j} e^{+jk\hat{k}\cdot\vec{r}_{m'}} \left(\mathbf{I} - \hat{k}\hat{k}\right) \cdot \vec{f}_j' dS' dA_{\hat{k}} \qquad (9)$$

with the abbreviation

$$T_\infty(\kappa;\cos\alpha)=\sum_{l=0}^{\infty}\frac{2l+1}{j^l}\,\mathrm{h}_l^{(2)}(\kappa)\,\mathrm{P}_l(\cos\alpha)\quad, \tag{10}$$

where P are the Legendre polynomial, and $\mathrm{h}^{(2)}$ the spherical Hankel function 2nd kind. The geometrical relations are elucidated in Fig. 2, k and Z_F are the free space wavenumber and the free space wave impedance, respectively.

Analogous expressions are derived for $\mathbf{Z}_2$, and for the matrices $\mathbf{T}$, which contain the eigenvectors of the apertures. The singularities have to be eliminated in the usual way, cf. e.g. [15].

In addition to the field integral equation, we still have to formulate the impedance relations on the waveguide apertures. From the continuity of the tangential magnetic field strength on the apertures

$$2\mathbf{H}_t^{\mathrm{inc}}+\mathbf{H}_t^r\left\{\mathbf{M}_S\right\}=\mathbf{J}_S\quad\hat{n}, \tag{11}$$

and with the sum expressions containing the modal eigenvectors $\mathbf{h}$ of the forward directed (p) and reflected (r) wave terms

$$2a_i\mathbf{h}_i^p+\sum_j b_j\mathbf{h}_j^r=\sum_k \mathbf{J}_k\vec{f}_k\quad\hat{n} \tag{12}$$

we obtain with $\mathbf{h}_i^p=-\mathbf{h}_i^r$, after scalar multiplication with $\vec{g}_i=\mathbf{e}_i^r\quad\hat{n}$ and integration over the corresponding aperture areas,

$$\sum_k J_k\iint\mathbf{e}_i^r\cdot\vec{f}_k dS-\sum_j b_j\iint(\mathbf{e}_i^r\quad\hat{n})\cdot\mathbf{h}_j^r dS= \\ -2a_i\iint(\mathbf{e}_i^r\ .\ \hat{n})\cdot\mathbf{h}_i^r dS \tag{15}$$

This impedance relation contains the modal excitation term for an excitation with the i-th waveguide mode. The equation is correspondingly weighted and added to the lower part of the EFIE.

III. Results

The first example is a single circular horn with peripheral choke (Fig.3) calculated including the outer geometry. The MLFMA is carried out in two levels, contains 370 groups, and 4500 unknowns. The storage requirement is about 115 Mbyte; the calculation time for the farfield pattern is only a few seconds on a 2 GHz P4 PC. The MLFMA results are in good agreement with the standard MoM solution.

Three radiating rectangular apertures according to Bird [5] but within a finite plate are shown in Fig. 4. For the MLFMA, 10325 unknowns, 336 groups, 3 levels have been considered. A Cholesky pre-conditioner has been applied. The storage requirement was 450 MB. Good agreement with S-parameter measurements reported in [5] can be stated.

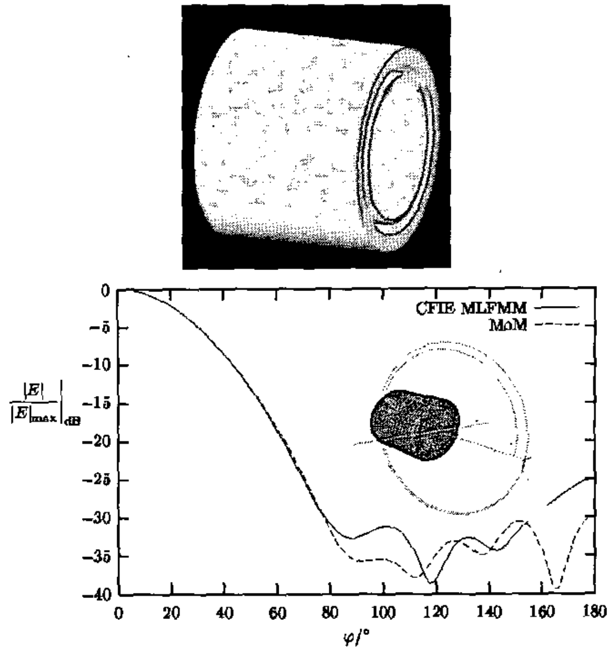

Fig. 3. Choked aperture, inner radius 11.5mm, outer radius 17mm, choke depth 7.5mm, width 1.5mm, distance from inner circular waveguide 1mm, frequency 15.65 GHz

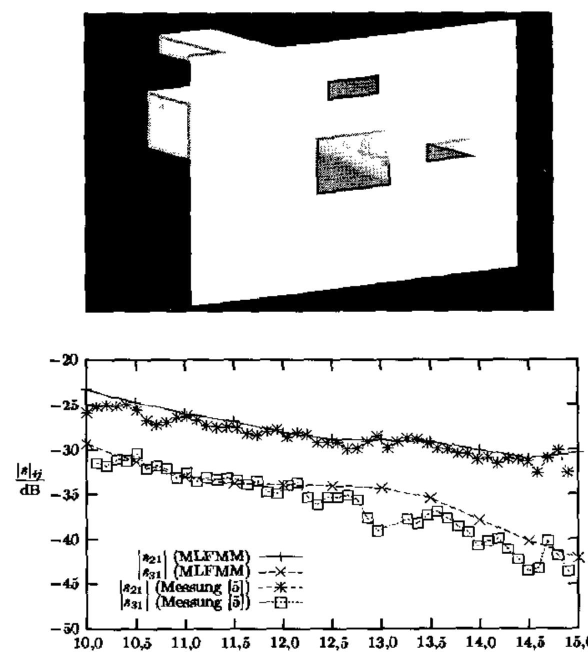

Fig. 4. Three rectangular apertures according to [5], but in finite rectangular plate of size 100mm x 100mm. Aperture sizes: 22.8mm² center aperture, 15.7mm x 7.7mm lateral apertures, displacement 30mm, f= 12.5 GHz

The next example is a horn cluster consisting of four circular apertures in a finite circular screen (Fig.5). All apertures are assumed excited with their fundamental H^c_{11} modes, with same amplitude and phase. For the MLFMA, 12772 unknowns, 1616 groups, 4 levels have been considered. 6 modes are taken into account in each aperture. The storage requirement was 365 MB. The E-field pattern shows the comparison between the EFIE and combined field integral equation (CFIE) MLFMA solution. Within the main beam range $\varphi = -180° \ldots -100°,\ 100° \ldots 180°$, respectively, good agreement can be stated, that is, the simpler EFIE MLFMA can be used. Behind the aperture cluster, however, the EFIE MLFMA results are not accurate, as the screen has to be left open for the EFIE calculations in order to avoid the known internal resonances when applying the EFIE for closed structures. For this problem the CFIE MLFMA has to be applied.

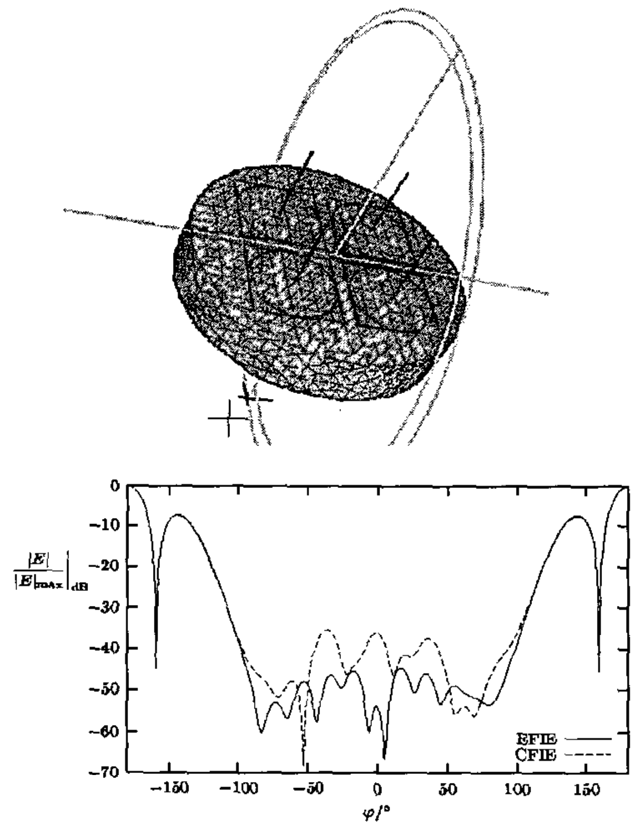

Fig. 5. Four circular apertures in a finite circular metallic screen of diameter 50mm. Aperture diameters 15mm, center distances 17.5mm. Frequency 24 GHz. EFIE and CFIE MLFMA comparison.

IV. Conclusion

An extended multilevel fast multipole algorithm (MLFMA) applied for the rigorous analysis of coupled rectangular and circular apertures of different size within a finite conducting screen leads to reduced computational efforts for this class of challenging problems. The utilization of aperture waveguide modal eigenvectors and Rao-Wilton-Glisson (RWG) functions for triangular patches as basis functions for the magnetic and electric surface current densities, respectively, yields stable and reliable results. A combined field intregral equation (CFIE) MLFMA avoids internal resonances. The method is verified by available measurements and reference calculations.

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